Erratum of "Modernizing Markov chains Monte Carlo for Scientific and Bayesian Modeling"

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Posterior draws for latent Gaussian variables

Algorithm 5.5, for generating posterior draws, θ^*, in a latent Gaussian models has an error.

For θ^*, we need to examine the "extended" prior covariance matrix,

$$\mathbf{K} = \begin{bmatrix} K(X,X) & K(X,X^*) \\ K(X,X^*) & K(X^*,X^*) \end{bmatrix}.$$

I'll denote $K = K(X,X)$, $K^* = K(X,X^*)$ and $K^{**} = K(X^*,X^*)$. We can abstract the problem further, with

$$\mathbf{K} = \begin{bmatrix} K & K^* \\ K^* & K^{**} \end{bmatrix}$$

where the different components of K are arbitrarily specified (i.e. they're not the result of one fixed covariance function applied to different X's).

The error is in the specification of the approximate covariance matrix for $\pi_{\mathcal{G}}(\theta^* \mid y, \phi, \eta)$. The correct covariance is

$$\Sigma_{\mathcal{G}}(\theta^* \mid X, y, \phi, X^*) = \Sigma_{\mathcal{G}}(\theta^* \mid y, \phi, \eta, X, X^*) = K^{**} - K^*(K + W^{-1})^{-1}K^*.$$

The thesis incorrectly stipulates that the first term is K^*, rather than K^{**}.

Accordingly, Algorithm 5.5 should be revised to produce the following

Algorithm 1: Posterior draws for latent Gaussian θ^*

intpput: $y, \phi, \eta, X, X^*, K(\phi, X, X^*), \pi(y \mid \theta, \eta)$

saved input from the Newton solver: $\hat{\theta}$, W, K, $\nabla_{\hat{\theta}} \log \pi(y \mid \hat{\theta}, \eta)$

  $W^{\frac{1}{2}}$, L ▷ $B = I + W^{\frac{1}{2}} K W^{\frac{1}{2}}$, $LL^T = B$

  W, $K^{\frac{1}{2}}$, L ▷ $B = I + K^{\frac{1}{2}T} W K^{\frac{1}{2}}$, $LL^T = B$

  W, L, U ▷ $B = I + KW$, $LU = B$

$K^* \leftarrow K(X, X^*)$

$K^{**} \leftarrow K(X^*, X^*)$

$\mu^* \leftarrow K^* \nabla_{\hat{\theta}} \log \pi(y \mid \theta, \eta)$

if $(B = I + W^{\frac{1}{2}} K W^{\frac{1}{2}})$ **then**

  $V \leftarrow L \backslash W^{\frac{1}{2}} K^*$

  $\Sigma^* \leftarrow K^{**} - V^T V$

else if $(B = I + K^{\frac{1}{2}T} W K^{\frac{1}{2}})$ **then**

  $D \leftarrow L \backslash K^{\frac{1}{2}} W$

  $R \leftarrow W - D^T D$

  $\Sigma^* \leftarrow K^{**} - K^* R K^*$

else if $(B = I + KW)$ **then**

  $\Sigma^* = K^{**} - K^*(W - WU \backslash L \backslash KW) K^*$

end

$\theta^* \sim \text{Normal}(\mu^*, \Sigma^*)$

return: θ^*.